Chaotic Inflation with ranning nonminimal conupling

著者	二間瀬　敏史
journal or publication title	Physical review. D
volume	60
number	6
page range	063511-1-063511-6
year	1999
URL	http://hdl.handle.net/10097/35594

doi: 10.1103/PhysRevD.60.063511

Chaotic inflation with a running nonminimal coupling

Toshifumi Futamase
Astronomical Institute, Tohoku University, Sendai, 980-77 Japan

Masahiro Tanaka
Department of Physics, Ochanomizu University, Tokyo, 112 Japan
and R&D Center, Kyocera Corporation, Kagoshima, 899-4312 Japan

(Received 14 April 1997; published 20 August 1999)

We find a successful model of chaotic inflation with an inflaton coupled nonminimally with gravity where the coupling constant ξ runs with the evolution of the inflaton. The running of the coupling is slow enough for natural values of the Yukawa couplings. If a small value of ξ is chosen at any given scale M, then the coupling remains small enough to have a sufficient period of inflation. [S0556-2821(99)06812-5]

PACS number(s): 98.80.Cq, 11.10.Gh

I. INTRODUCTION

Chaotic inflation [1,2] is known to be a successful scenario of inflation. Although its setup is regarded as simpler than other scenarios, it still needs fine-tuning of some parameters, as others do. The major two are the inflaton self-coupling constant [2], and the nonminimal coupling constant to gravity [3–6].

As far as the former is concerned, we may identify the direction which contains only quadratic terms, e.g., a mass term, with an inflaton and then we can avoid fine-tuning [7]. The direction is called the flat direction,[1] which frequently appears in supersymmetric models. Interactions still exist in the theory but such terms do not appear along the flat direction.

Unfortunately so far no mechanism is known to avoid the latter fine-tuning. We have the freedom to specify how the inflaton couples with gravity, namely how to choose the nonminimal coupling constant. The nonminimal coupling with gravity leads to a soft supersymmetry (SUSY) breaking [8] and is needed for renormalization. In the case of positive nonminimal coupling constant it has been shown that the chaotic inflationary scenario does not work unless it is sufficiently small [3–5]. On the other hand, the scenario does work in the case of negative coupling constant. In particular Fakir and Unruh [6] show that a model with a large negative nonminimal coupling constant gives the appropriate amplitude of density perturbation without making any fine-tuning for the self-coupling.

However, the previous studies on the chaotic inflation scenario with nonminimal coupling are restricted to the classical treatment. The aim of this paper is to take into account the quantum effect on the coupling in the context of chaotic inflation. The effect makes the coupling constant running with the inflaton field. The running nature allows us to have a new possibility which is not available in classical treatments. Namely, there are always some regions in spacetime where the coupling constant becomes small enough to have a sufficient period of inflation. We shall study this possibility in SUSY minded models and find that this is the case with a reasonable range of parameters.

We study a SUSY minded model in this paper. But we mean *global* SUSY instead of *local* SUSY simply by SUSY. Most papers on SUSY in curved space are based on supergravity[2] since we need to include graviton and its partner in order to maintain SUSY in curved space. Following are the reasons why we adopt *global* SUSY: we treat SUSY in curved space as softly broken global SUSY, therefore need not localize SUSY; we can deal with a renormalizable theory.

This paper is organized as follows. In Sec. II we introduce our model which is the bosonic sector of a SUSY model in curved spacetime. Then we shall give an expression for the running coupling constant in Sec. III. In Sec. IV we clarify the condition on Yukawa coupling for the realization of the chaotic inflationary scenario. Finally some discussions are given in Sec. V.

II. CHAOTIC INFLATION WITH NONMINIMAL COUPLING TO GRAVITY

According to the above argument, we shall take the following action for our model of the nonminimally coupled inflaton in Einstein gravity [9,10]:

$$S=-\frac{1}{2}\int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{8\pi}\mathcal{R}+\partial_\mu\phi\cdot\partial^\mu\phi+m^2(\phi)\phi^2-\xi(\phi)\mathcal{R}\phi^2\right], \tag{1}$$

[1]We mean the direction without interaction terms by flat direction. We assume that the non-renormalizable terms are also vanishing because the inflaton will be the scalar field with the flattest potential.

[2]Supergravity models which many authors study are the conformal transformed theory to obtain the Einstein one ($\xi=0$). But conformal symmetry is anomalous quantum mechanically. Then the Einstein theory is nonequivalent to its conformally related one, although we have no reason to choose the former. Contrary to them, our model has arbitrary nonminimal coupling.

where $M_{Pl}=1/\sqrt{G}$ and $m(\phi)$ and $\xi(\phi)$ are the renormalized mass and the renormalized nonminimal coupling constant, respectively. Conformal invariance yields $\xi(\phi)=1/6$. If the parameters do not receive any renormalization, they are ϕ-independent as we shall see in Sec. III. The above action is considered to be the bosonic sector of a Wess-Zumino model in curved spacetime and the Einstein term. The nonminimal coupling is a soft SUSY breaking term as mass term. The equations of motion are

$$[\Box-\overline{m}^2(\phi)+\overline{\xi}(\phi)\mathcal{R}]\phi=0, \tag{2}$$

$$\mathcal{R}_{\mu\nu}-\frac{1}{2}\mathcal{R}g_{\mu\nu}=\frac{8\pi}{M_{Pl}^2}\mathcal{T}_{\mu\nu}^{eff}, \tag{3}$$

where

$$\overline{m}^2(\phi)=m^2(\phi)+\frac{1}{2}\phi\frac{dm^2(\phi)}{d\phi}, \tag{4}$$

$$\overline{\xi}(\phi)=\xi(\phi)+\frac{1}{2}\phi\frac{d\xi(\phi)}{d\phi}, \tag{5}$$

$$\mathcal{T}_{\mu\nu}^{eff}=\frac{M_{Pl}^2}{M_{Pl}^2-8\pi\xi(\phi)\phi^2}\Bigg[\{1-2\xi(\phi)\}\partial_\mu\phi\cdot\partial_\nu\phi+\left\{2\xi(\phi)-\frac{1}{2}\right\}\partial_\rho\phi\cdot\partial^\rho\phi g_{\mu\nu} -2\xi(\phi)\phi\nabla_\mu\partial_\nu\phi-\frac{1}{2}m^2(\phi)\phi^2 g_{\mu\nu}-2\xi(\phi)g_{\mu\nu}\partial^\rho\phi\cdot\partial_\rho\phi\Bigg]. \tag{6}$$

Since we focus on small regions in the spacetime in which the inflaton distributes homogeneously and examine if such a region will undergo sufficient period of the inflation, we shall assume that the region is homogeneous and isotropic, and thus can be described by the Robertson-Walker metric. Then we have the Friedmann and Raychaudhuri equation for such a region:

$$H^2+\frac{k}{a^2}=\frac{1}{6}\cdot\frac{\dot\phi^2+12\xi(\phi)H\dot\phi\phi+m^2(\phi)\phi^2}{M_{Pl}^2/8\pi-\xi(\phi)\phi^2}, \tag{7}$$

$$\dot H+H^2=\frac{1}{6}\cdot\frac{\{6\xi(\phi)-2\}\dot\phi^2+m^2(\phi)\phi^2+6\xi(\phi)\ddot\phi\phi+6\xi(\phi)H\dot\phi\phi}{M_{Pl}^2/8\pi-\xi(\phi)\phi^2}, \tag{8}$$

where $H=\dot a/a$ is the Hubble parameter of the region, with a the scale factor. Inserting the Eqs. (7) and (8) into Eq. (2), we obtain the Klein-Gordon type equation for the inflaton:

$$\ddot\phi+3H\dot\phi+\left[\overline{m}^2(\phi)+\frac{\{6\xi(\phi)-1\}\dot\phi^2+2m^2(\phi)\phi^2+6\xi(\phi)\{\ddot\phi+3H\dot\phi\}\phi}{M_{Pl}^2/8\pi-\xi(\phi)\phi^2}\overline{\xi}(\phi)\right]\phi=0. \tag{9}$$

For a negative constant ξ it is known that there are the two saddle points in the phase space, where $\phi_{c-}=\pm M_{Pl}/\sqrt{8\pi|\xi|}$ and $\dot\phi_{c-}=0$ [9,10]. In our case this is modified as

$$\phi_{c-}=\pm\frac{M_{Pl}\overline{m}(\phi)}{\sqrt{8\pi\{\overline{m}^2(\phi)\xi(\phi)-2m^2(\phi)\overline{\xi}(\phi)\}}}, \tag{10}$$

$$\dot\phi_{c-}=0, \tag{11}$$

where $\overline{m}^2(\phi)\xi(\phi)-2m^2(\phi)\overline{\xi}(\phi)>0$. This is easily understood if we insert it to Eq. (9). The origin is the unique stable point in the phase space. If the inflaton initially satisfies the condition

$$-\frac{M_{Pl}\overline{m}(\phi)}{\sqrt{8\pi\{\overline{m}^2(\phi)\xi(\phi)-2m^2(\phi)\overline{\xi}(\phi)\}}}<\phi<\frac{M_{Pl}\overline{m}(\phi)}{\sqrt{8\pi\{\overline{m}^2(\phi)\xi(\phi)-2m^2(\phi)\overline{\xi}(\phi)\}}}, \tag{12}$$

chaotic inflation may occur. Otherwise, the scalar field keeps growing exponentially: inflation occurs but never terminates [9].

On the other hand, for a positive ξ the anisotropic shear diverges as the inflaton approaches at the following points without bound [4,11]:

$$\phi_{c+} = \pm \frac{M_{Pl}}{\sqrt{8\pi\xi(\phi)}}. \tag{13}$$

The evolution of the region with ϕ larger than $\phi_{c+}(\phi)$ will terminate at $\phi_{c+}(\phi)$, and such regions do not evolve into our present Universe. We shall call these points the critical points. Thus we shall only pay attention to the region with ϕ lying between the two critical points [3–5].

It has been known that the minimal initial value of the inflaton is about $5M_{Pl}$ to realize sufficient period of inflation in the framework of chaotic inflationary scenario. Then the above conditions (12) and (13) with $|\phi| \sim 5M_{Pl}$ give us the condition $|\xi| \simeq 10^{-3}$ for successful chaotic inflationary scenario with nonminimal coupling. In Ref. [3] they demand also more severe constraints for natural realization of inflation, because they consider that inflaton probably has Planck energy density initially. We do not adopt such a view; instead we assume that the initial value is distributed randomly below Planck energy density. Therefore we just need $|\phi| \gtrsim 5M_{Pl}$ for the initial condition.

III. RUNNING NONMINIMAL COUPLING

We adopt the one-loop effective action along a flat direction of the Wess-Zumino model [12] as the Lagrangian (1). The superpotential is $W = g\Phi_1\Phi_2\Phi_2/2$, where Φ_1 and Φ_2 are superfields [13]. The action is written as

$$e^{-1}\mathcal{L} = -(\partial_\mu \phi_i)^*(\partial^\mu \phi_i) - V(\phi_1, \phi_2) + \frac{1}{2}(\overline{\psi_1}\,\overline{\psi_2})[i\mathcal{D} + M(\phi_1,\phi_2)]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \tag{14}$$

where $\mathcal{D}_\mu$ is covariant derivative and the potential term V is defined as

$$V(\phi_1,\phi_2) = (m_1^2 - \xi_1\mathcal{R})\phi_1^*\phi_1 + \frac{1}{4}|g^2|(\phi_2^*\phi_2)^2 + (m_2^2 - \xi_2\mathcal{R})\phi_2^*\phi_2 + |g^2|(\phi_1^*\phi_1)(\phi_2^*\phi_2), \tag{15}$$

and the mass matrix $M(\phi_1,\phi_2)$ is given by

$$M(\phi_1,\phi_2) = \begin{pmatrix} 0 & \mathrm{Re}(g\phi_2) - \mathrm{Im}(g\phi_2)\gamma^5 \\ \mathrm{Re}(g\phi_2) - \mathrm{Im}(g\phi_2)\gamma^5 & \mathrm{Re}(g\phi_1) - \mathrm{Im}(g\phi_1)\gamma^5 \end{pmatrix}. \tag{16}$$

$\mathcal{R} = 12H^2$ is the scalar curvature in de Sitter spacetime . In the scalar potential (14), we have introduced the soft SUSY breaking mass terms $m_i^2\phi_i^*\phi_i$ and the nonminimal curvature couplings $\xi_i\mathcal{R}\phi_i^*\phi_i$, in addition to the minimal extension of the Wess-Zumino model in curved spaces. In this paper we mean *global* SUSY instead of *local* SUSY (supergravity) simply by SUSY. Then we can study a *renormalizable* model. Since the nonminimal coupling receives the renormalization as will be seen below, the bare term $\xi_i\mathcal{R}\phi_i^*\phi_i$ is necessary for this model to be renormalizable. From the tree potential (15), we see $\phi_2 = 0$ is actually a flat direction in this model; namely the potential energy remains flat for any values of ϕ_1 except for the SUSY breaking mass term and the nonminimal coupling term.

Now let us consider the one-loop effective potential. We decompose the scalar fields as

$$\phi_1 \equiv \frac{\phi_1}{\sqrt{2}} + \varphi_1 + i\varphi_2, \quad \phi_2 \equiv \frac{\phi_2}{\sqrt{2}} + \varphi_3 + i\varphi_4, \tag{17}$$

where all the fields are real and ϕ_i are the classical fields.

Using the DeWitt-Schwinger technique [14] we obtain

$$\begin{aligned} V_{eff}(\phi_1,\phi_2) = {} & \frac{1}{2}(m_1^2 - \xi_1\mathcal{R})\phi_1^2 + \frac{1}{2}(m_2^2 - \xi_2\mathcal{R})\phi_2^2 \\ & + \frac{1}{16}g^2\phi_2^4 + \frac{1}{4}g^2\phi_1^2\phi_2^2 + \frac{g^2}{32\pi^2}\ln\frac{\phi_1^2 + \cdots}{\Lambda^2} \\ & \times\left[\left\{m_2^2 + \left(\xi_2 - \frac{1}{4}\right)\right\}(\phi_1^2 + \phi_2^2) + \frac{1}{4}g^2\phi_1^2\phi_2^2 \right. \\ & \left. + \frac{1}{16}g^2\phi_2^4\right] + \frac{g^2}{32\pi^2}\ln\frac{\phi_2^2 + \cdots}{\Lambda^2} \\ & \times\left[\left\{m_1^2 + \left(\xi_1 - \frac{1}{4}\right)\right\}\phi_2^2 + \frac{1}{4}g^2\phi_1^2\phi_2^2\right] \end{aligned} \tag{18}$$

in the flat limit, where we have omitted the ϕ_i-independent convergent terms which are irrelevant to our present analysis. $(\cdots)$ do not contain any ϕ_1-dependent term. Likewise [15,16], the renormalized wave functions are defined as

$$\phi_{1R}^2 = \left(1 + \frac{g^2}{32\pi^2}\ln\frac{\Lambda^2}{\phi_1^2 + \cdots}\right)\phi_1^2, \tag{19}$$

$$\phi_{2R}^2=\left(1+\frac{g^2}{32\pi^2}\ln\frac{\Lambda^2}{\phi_1^2+\cdots}+\frac{g^2}{32\pi^2}\ln\frac{\Lambda^2}{\phi_2^2+\cdots}\right)\phi_2^2. \tag{20}$$

Then we define the renormalized parameters

$$g_R^2=\left(1+\frac{3g^2}{32\pi^2}\ln\frac{\phi_1^2+\cdots}{\Lambda^2}\right)g^2, \tag{21}$$

$$m_{1R}^2=m_1^2+\frac{g^2}{32\pi^2}\left(\ln\frac{\phi_1^2+\cdots}{\Lambda^2}\right)(m_1^2+2m_2^2), \tag{22}$$

$$m_{2R}^2=m_2^2+\frac{3g^2}{32\pi^2}\left(\ln\frac{\phi_1^2+\cdots}{\Lambda^2}\right)m_2^2, \tag{23}$$

$$\xi_{1R}=\xi_1+\frac{g^2}{32\pi^2}\left(\ln\frac{\phi_1^2+\cdots}{\Lambda^2}\right)\left(\xi_1+2\xi_2-\frac{1}{2}\right), \tag{24}$$

$$\xi_{2R}=\xi_2+\frac{g^2}{32\pi^2}\left(\ln\frac{\phi_1^2+\cdots}{\Lambda^2}\right)\left(3\xi_2-\frac{1}{2}\right), \tag{25}$$

where we present only correction with ϕ_1 (the flat direction). The renormalizaiton groups are

$$\phi\frac{dg^2}{d\phi}=\frac{3g^4}{16\pi^2}, \tag{26}$$

$$\phi\frac{dm_1^2}{d\phi}=\frac{g^2}{16\pi^2}(m_1^2+2m_2^2), \tag{27}$$

$$\phi\frac{dm_2^2}{d\phi}=\frac{3g^2}{16\pi^2}m_2^2, \tag{28}$$

$$\phi\frac{d\xi_1}{d\phi}=\frac{g^2}{16\pi^2}\left(\xi_1+2\xi_2-\frac{1}{2}\right), \tag{29}$$

$$\phi\frac{d\xi_2}{d\phi}=\frac{3g^2}{16\pi^2}\left(\xi_2-\frac{1}{6}\right), \tag{30}$$

where we simply denote ϕ_1 by ϕ and hereafter all the quantities are renormalized. The renormalization group analysis of the nonminimal coupling says

$$\xi_2(\phi)=\frac{g^2(\phi)}{g^2(M)}\left[\xi_2(M)-\frac{1}{6}\right]+\frac{1}{6}, \tag{31}$$

where

$$g^2(\phi)=g^2(M)\left[1-\frac{3g^2(M)}{32\pi^2}\ln\frac{\phi^2}{M^2}\right]^{-1}. \tag{32}$$

Equations (29) and (30) allow us to assume $\xi_1(\phi)=\xi_2(\phi)\equiv\xi(\phi)$. This theory is free and conformal invariant in the infrared limit; $\phi\to 0$.[3] In other words, in this limit the free scalar field propagates along the light cone [5]. The form of Eq. (31) is universal regardless of whether its conformal invariance appears in the ultraviolet or infrared region [17,18].[4]

As far as $g^2(\phi)<1$ and $3g^2(M)\ln(\phi^2/M^2)/32\pi^2<1$, one-loop calculation is reliable.

Contrary to m, identification of the inflaton with some elementary particle cannot constrain ξ. If we define the renormalization point M as

$$\xi(M)=0, \tag{33}$$

then Eq. (32) becomes

$$\xi(\phi)=\frac{1}{6}-\frac{1}{6}\left[1-\frac{3g^2(M)}{32\pi^2}\ln\frac{\phi^2}{M^2}\right]^{-1}. \tag{34}$$

We use the Eq. (34)[5] for the later argument, where we assume Eq. (34) is applicable up to the Planck energy density: $\phi\simeq M_{Pl}^2/m$.

IV. CONDITION ON YUKAWA COUPLING

In this section we clarify the condition on the Yukawa coupling constant for the realization of a successful inflationary scenario using the above results. We shall show that the reason we have a small ξ is not for our appropriate choice of the value of M. In fact we examine a wide variety of M and show the existence of the reasonable range of parameters to have a small ξ and its slow running during the inflation.

First of all we assume that the inflaton must have the initial value [1–3]:[6]

$$\phi\gtrsim 5M_{Pl}. \tag{35}$$

As discussed in Sec. II this is needed for a sufficient period of inflation. Since the two types of critical points Eqs. (10) and (13) have the crossing, it is convenient to divide the range of M as $5M_{Pl}<\phi_{crs}$ and $\phi_{crs}\lesssim 5M_{Pl}$, which we call

[3]The running of the mass parameter is $m_2^2(\phi)=\{g^2(\phi)/g^2(M)\}m_2^2(M)$. If we choose a right-handed sneutrino [7] as the inflaton, it is plausible to assume $m(M_{Pl})\simeq 10^{13}$ GeV. $m(\phi)$ does not change its order of magnitude drastically in the relevant scale.

[4]What we actually do is called renormalization group improvement of effective potential [18–20].

[5]In other words we choose the flow which decreases monotonically to the negative infinity as $3g^2(M)\ln(\phi^2/M^2)/32\pi^2\to 1$.

[6]Hereafter we concentrate on the positive side of ϕ.

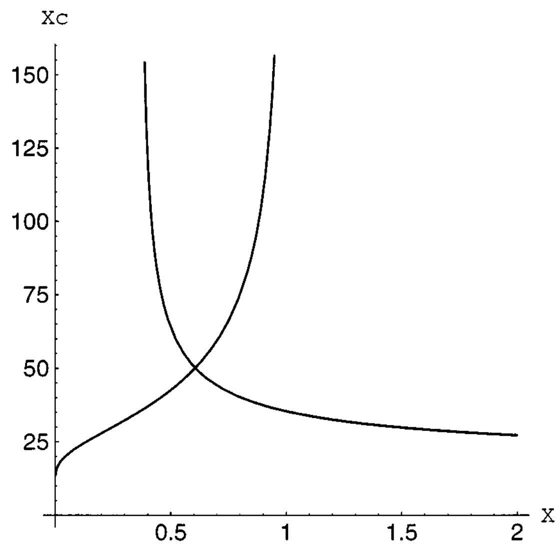

FIG. 1. $x \equiv \phi/M$, $x_c \equiv \phi_{c\pm}/M_{Pl}$, and $g^2(M)=10^{-2}$. The monotonically increasing function is ϕ_{c+} and the decreasing is ϕ_{c-}.

case (I) and case (II), respectively. Since Eqs. (27) and (28) say $m^2(\phi) \simeq \overline{m}^2(\phi)$ in the perturbation theory, Eq. (10) is reduced to

$$\phi_{c-} = \pm \frac{M_{Pl}}{\sqrt{8\pi\{\xi(\phi) - 2\overline{\xi}(\phi)\}}}. \tag{36}$$

The crossing is characterized by the equation

$$-2\overline{\xi}(\phi_{crs}) + \xi(\phi_{crs}) = \xi(\phi_{crs}). \tag{37}$$

Then we obtain

$$\ln \frac{\phi_{crs}^2}{M^2} = \frac{1 \pm \sqrt{1+2\alpha}}{\alpha}, \tag{38}$$

where $\alpha = 3g^2(M)/16\pi^2$. In the range where the perturbation is reliable we obtain

$$\phi_{crs} \simeq \frac{M}{\sqrt{e}}. \tag{39}$$

(See Fig. 1.)

A. Case (I)

In this case $\xi(5M_{Pl})$ is positive. Then during evolution of inflaton $\xi(\phi)$ is always positive. Since all we must to do is to avoid one of the critical points, ϕ_{c+}, we obtain the condition

$$\frac{M_{Pl}}{\sqrt{8\pi\xi(\phi)}} > \phi \tag{40}$$

during the whole evolution of ϕ, $\phi \lesssim 5M_{Pl}$.

The above condition is rewritten as (see Fig. 2)

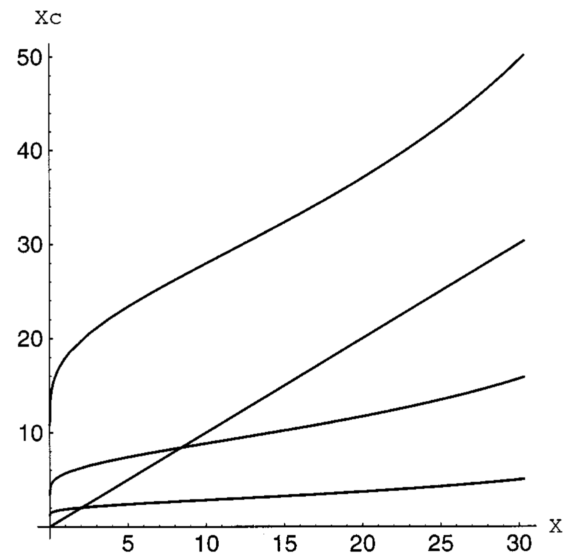

FIG. 2. $x \equiv \phi/M_{Pl}$, $x_c \equiv \phi_{c\pm}/M_{Pl}$, and $M/M_{Pl}=50$. The upper line corresponds to the case $g^2(M)=10^{-2}$, the middle line to the case $g^2(M)=10^{-1}$, and the lower to the case $g^2(M)=1$. The straight line indicates $x_c = x$: $\phi_{c\pm}=\phi$. The former two lead to a successful inflation because $\phi_{c\pm} > \phi$ for $\phi < 5M_{Pl}$.

$$\frac{M_{Pl}}{2}\sqrt{\frac{3}{\pi} - \frac{32\pi}{g^2(M)\ln(\phi/M)^2}} > \phi \tag{41}$$

for all $\phi \lesssim 5M_{Pl}$.

Numerical results say that for $10 \lesssim M/5M_{Pl} < 10^2$ we need $g^2(M) \lesssim 1$ and for $10^2 \lesssim M/5M_{Pl} \lesssim 10^6$ we need $g^2(M) \lesssim 10^{-1}$. We will not consider the range $M/5M_{Pl} > 10^6$ because the energy density becomes larger than Planck energy density for such ranges.

B. Case (II)

In this case $\xi(5M_{Pl})$ is small or negative. $5M_{Pl}$ should be smaller than ϕ_{c-} and during the subsequent evolution ϕ must avoid ϕ_{c+}:

$$\frac{M_{Pl}}{\sqrt{8\pi\{\xi(5M_{Pl}) - 2\overline{\xi}(5M_{Pl})\}}} > 5M_{Pl}, \tag{42}$$

$$\frac{M_{Pl}}{\sqrt{8\pi\xi(\phi)}} > \phi \tag{43}$$

for all $\phi < M$. Equation (42) is sufficient for $\phi > M$ since $\phi_{c-}(\phi)$ monotonically decreases in such a region.

Equation (42) gives the condition on ξ as

$$\xi(5M_{Pl}) - 2\overline{\xi}(5M_{Pl}) \lesssim 10^{-3}, \tag{44}$$

which is the same as the condition for the constant ξ [3]. Roughly this gives us the following condition on $g(M)$ (see Fig. 3):

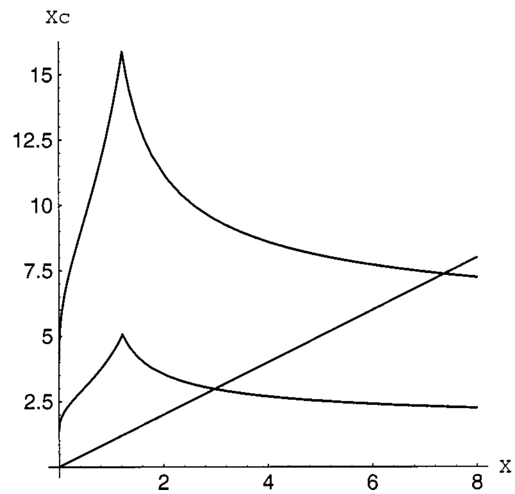

FIG. 3. $M/M_{Pl}=2$. The upper line corresponds to the case $g^2(M)=10^{-1}$ and the lower does to the case $g^2(M)=1$. The straight line indicates $x_c=x$: $\phi_{c\pm}=\phi$ The former leads to a successful inflation because $\phi_{c\pm}>\phi$ for $\phi<5M_{Pl}$. From Eq. (45) we can find with this value of M we need $g^2(M)<0.26$ approximately.

$$g^2(M)\lesssim\frac{1}{2+\ln(5M_{Pl}/M)^2}. \tag{45}$$

This gives us the condition on $g(M)$. For example, $g^2(M) \sim 10^{-1}$ for $M/5M_{Pl}\sim 10^{-2}$, $g^2(M)\sim 10^{-2}$ for $M/5M_{Pl} \sim 10^{-22}$ and so on. We can see the condition is not unreasonable at all in SUSY models (see, for example Ref. [7]).

The second condition (43) is automatically satisfied if perturbation is reliable: $g(\phi)<1$ (see Fig. 3).

V. CONCLUSIONS AND DISCUSSION

We have found that chaotic inflation by nonminimal coupled inflaton is naturally realized by taking into account the running nature of the coupling constant. The condition to have a successful inflation is just $g(M)\lesssim 10^{-1}$ around a wide range of M where $\xi(M)=0$, which is not unreasonable in some models. It also implies that efficient reheating may be possible. Nonminimal coupling constant becomes small enough ($|\xi|\lesssim 10^{-3}$) when inflation starts and evolves to the conformal value, where the universe is radiation dominant.

The above argument is true to all the models which have flat directions since the form of the renormalization group equation is universal.

A model with $-\xi\gtrsim 10^4$ is considered to circumvent fine-tuning of the self-coupling constant in $\lambda\phi^4$ theory [6]. But such a large $|\xi|$ is realized by a large ϕ which causes very high energy density larger than the Planck energy density. Accordingly, such a model seems to be not so feasible. However, the more detailed investigation is necessary to draw any definite conclusion for such cases.

ACKNOWLEDGMENTS

The authors thank M. Hotta and K. Kumekawa for valuable comments. They are grateful to T. Muta and S.D. Odintsov for their contribution to this revised manuscript.

[1] A. Linde, Phys. Lett. **129B**, 177 (1983).
[2] For a review, see A. Linde, *Particle Physics and Inflationary Cosmology* (Harwood Academic, Chur, Switzerland, 1990).
[3] T. Futamase and K. Maeda, Phys. Rev. D **39**, 399 (1989).
[4] T. Futamase, T. Rothman, and R. Matzner, Phys. Rev. D **39**, 405 (1989).
[5] V. Faraoni, Phys. Rev. D **53**, 6813 (1996).
[6] R. Fakir and W. G. Unruh, Phys. Rev. D **41**, 1783 (1990); N. Makino and M. Sasaki, Prog. Theor. Phys. **86**, 103 (1991).
[7] H. Murayama, H. Suzuki, T. Yanagida, and J. Yokoyama, Phys. Rev. Lett. **70**, 1912 (1993).
[8] M. Tanaka, hep-th/9701063.
[9] L. Amendola, M. Litterio, and F. Occhionero, Int. J. Mod. Phys. A **5**, 3861 (1990).
[10] A. Barroso, J. Casasayas, P. Crawford, P. Moniz, and A. Nunes, Phys. Lett. B **275**, 264 (1992).
[11] A. A. Starobinsky, Pis'ma Astron. Zh. **7**, 67 (1981) [Sov. Astron. Lett. **7**, 36 (1981)].
[12] H. Suzuki and M. Tanaka, Phys. Rev. D **49**, 6692 (1994).
[13] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton Series in Physics, Princeton, 1982).
[14] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
[15] R. D. Ball, Phys. Rep. **182**, 1 (1989).
[16] M. Tanaka, Prog. Theor. Phys. **92**, 1105 (1994).
[17] I. L. Buchbinder and S. D. Odintsov, Yad. Fiz. **40**, 1338 (1984) [Sov. J. Nucl. Phys. **40**, 850 (1984)]; Lett. Nuovo Cimento **42**, 379 (1985).
[18] For a review, see I. L. Buchbinder, S. D. Odintsov, and I. L. Shapiro, *Effective Action in Quantum Gravity* (IOP, Bristol, 1992).
[19] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).
[20] I. L. Buchbinder and S. D. Odintsov, Class. Quantum Grav. **2**, 721 (1985).